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Dapatkan Penyelesaian :

1. $y''' - 3y'' + 9y' + 13y = 0$

Anggap penyelesaian : $y = e^{mx}$

Maka, akan didapatkan :

$y' = me^{mx}$

$y'' = m.(me^{mx})$

$= m^2e^{mx}$

$y''' = m^2(me^{mx})$

$= m^3e^{mx}$

1 ▶ Substitusi ke persamaan :

$m^3e^{mx} - 3(m^2e^{mx}) + 9\ me^{mx} + 13e^{mx} = 0$

$\underset{\text{pasti} \neq 0}{e^{mx}} [m^3 - 3m^2 + 9m + 13] = 0$

pasti $\neq 0$, maka : $m^3 - 3m^2 + 9m + 13 = 0$

2 Akar persamaan $m^3 - 3m^2 + 9m + 13 = 0$

• $m = -1 \Rightarrow -1 - 3 - 9 + 13 = 0$

$m = -1$ adalah salah satu akar persamaan.

Akar lain :

$$\begin{array}{r|rrrr} & 1 & -3 & 9 & 13 \\ -1 & & -1 & 4 & -13 \\ \hline & 1 & -4 & 13 & 0 \end{array} \Rightarrow m^2 - 4m + 13 = 0$$

Cari dengan rumus abc :

$$m_1, m_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$= 2 \pm \tfrac{1}{2}\sqrt{-36}$$

$$= 2 \pm \tfrac{1}{2}.6\sqrt{-1}$$

$$= \underset{a}{2} \pm \underset{b}{3}i$$

3 Karena $m_{1,2}$ kompleks, maka penyelesaiannya adalah

$y = e^{ax}[C_1 \cos(bx) + C_2 \sin(bx)]$

$= e^{2x}[C_1 \cos 3x + C_2 \sin 3x]$

Jadi, penyelesaiannya adalah $y = e^{2x}[C_1 \cos 3x + C_2 \sin 3x]$

2. $y''' - 3y'' + 3y' - y = 36e^x \ln x$

Penyelesaian

▶ Penyelesaian Umum PD Homogen : $y''' - 3y'' + 3y' - y = 0$

Substitusi misal penyelesaian : $y = e^{mx}$

Catatan : $y = e^{mx}$ $\quad y'' = m^2e^{mx}$

$y' = me^{mx}$ $\quad y''' = m^3e^{mx}$

Maka diperoleh : $y''' - 3y'' + 3y' - y = 0$

$m^3e^{mx} - 3[m^2e^{mx}] + 3[me^{mx}] - e^{mx} = 0$

$e^{mx}[m^3 - 3m^2 + 3m - 1] = 0$

Persamaan Karakteristik : $m^3 - 3m^2 + 3m - 1 = 0$

$(m-1)^3 = 0$

Jadi, penyelesaian berupa 3 akar real kembar (identik).

Sehingga, penyelesaian $\Rightarrow y(x) = e^{mx}[C_1 + C_2x + C_3x^2]$

$= e^x[C_1 + C_2x + C_3x^2]$

$= C_1e^x + C_2xe^x + C_3x^2e^x$

▶

memenuhi beberapa persamaan : $u_1'y_1 + u_2'y_2 + u_3'y_3 = 0$

$u_1'y_1' + u_2'y_2' + u_3'y_3' = 0$

$u_1'y_1'' + u_2'y_2'' + u_3'y_3'' = g(x)$

Dan diketahui bahwa :

$y_1 = e^x \qquad y_2 = xe^x \qquad y_3 = x^2e^x$

$y_1' = e^x \qquad y_2' = e^x + xe^x \qquad y_3' = 2xe^x + x^2e^x$

$y_1'' = e^x \qquad y_2'' = e^x + e^x + xe^x = 2e^x + xe^x = e^x(x+2) \qquad y_3'' = 2e^x + 2xe^x + 2xe^x + x^2e^x = 2e^x + 4x\,e^x + x^2e^x = e^x(x^2+4x+2)$

Persamaan :

$$\begin{aligned} u_1'(e^x) &+ u_2'(xe^x) &+ u_3'(x^2e^x) &= 0 \\ u_1'(e^x) &+ u_2'(e^x+xe^x) &+ u_3'(2xe^x+x^2e^x) &= 0 \\ u_1'(e^x) &+ u_2'(2e^x+xe^x) &+ u_3'(2e^x+4xe^x+x^2e^x) &= 36e^x\ln x \end{aligned}$$

atau bisa disederhanakan menjadi

$$\begin{aligned} u_1' &+ u_2'(x) &+ u_3'(x^2) &= 0 \\ u_1' &+ u_2'(x+1) &+ u_3'(x^2+2x) &= 0 \\ u_1' &+ u_2'(x+2) &+ u_3'(x^2+4x+2) &= 36\ln x \end{aligned}$$

Akan diperoleh :

$$W = \begin{vmatrix} 1 & x & x^2 \\ 1 & x+1 & x^2+2x \\ 1 & x+2 & x^2+4x+2 \end{vmatrix} \xrightarrow[-B_1+B_3]{-B_1+B_2} \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & 2x \\ 0 & 2 & 4x+2 \end{vmatrix} \xrightarrow{-2B_2+B_3} \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & 2x \\ 0 & 0 & 2 \end{vmatrix} = 2$$

$W = 2$

$$W_1 = \begin{vmatrix} 0 & x & x^2 \\ 0 & x+1 & x^2+2x \\ 36\ln x & x+2 & x^2+4x+2 \end{vmatrix} \xrightarrow{B_1 \leftrightarrow B_3} -1 \cdot \begin{vmatrix} 36\ln x & x+2 & x^2+4x+2 \\ 0 & x+1 & x^2+2x \\ 0 & x & x^2 \end{vmatrix}$$

$$\xrightarrow{-B_2+B_3} -1 \left| \begin{array}{ccc|cc} 36\ln x & x+2 & x^2+4x+2 & 36\ln x & x+2 \\ 0 & x+1 & x^2+2x & 0 & x+1 \\ 0 & -1 & -2x & 0 & -1 \end{array} \right|$$

dengan Sarus $:= -[36\ln x(x+1)(-2x) + 0 + 0 - 0 - 36\ln x(x^2+2x)(-1) - 0]$

$= -[36\ln x(-2x^2 - 2x + x^2 + 2x)]$

$= -[36\ln x(-x^2)]$

$W_1 = 36\ln x(x^2)$

$$W_2 = \begin{vmatrix} 1 & 0 & x^2 \\ 1 & 0 & x^2+2x \\ 1 & 36\ln x & x^2+4x+2 \end{vmatrix} \rightarrow \text{Sarus :} \left| \begin{array}{ccc|cc} 1 & 0 & x^2 & 1 & 0 \\ 1 & 0 & x^2+2x & 1 & 0 \\ 1 & 36\ln x & x^2+4x+2 & 1 & 36\ln x \end{array} \right|$$

$W_2 = 0 + 0 + x^2(36\ln x) - 0 - (x^2+2x)(36\ln x) - 0$

$= 36\ln x(x^2 - x^2 - 2x)$

$= -72x\ln x$

$$W_3 = \left|\begin{array}{ccc|cc} 1 & x & 0 & 1 & x \\ 1 & x+1 & 0 & 1 & x+1 \\ 1 & x+2 & 36\ln x & 1 & x+2 \end{array}\right|$$

$$\begin{aligned} W_3 &= (x+1)\,36\ln x + 0 + 0 - x(36\ln x) - 0 - 0 \\ &= 36\ln x\,(x+1-x) \\ &= 36\ln x \end{aligned}$$

$$L_1' = \frac{W_1}{W} = \frac{36x^2\ln x}{2} = 18x^2\ln x \qquad L_2' = \frac{W_2}{W} = \frac{-72x\ln x}{2} = -36x\ln x \qquad L_3' = \frac{W_3}{W} = \frac{36\ln x}{2} = 18\ln x$$

$\cdot\!> \int L_1'\,dx = \int 18x^2\ln x\,dx$

$=$

misal $=$ $u = \ln x$ $\quad dv = 18x^2 dx$

$du = \frac{1}{x}dx$ $\quad v = 6x^3$

$$\begin{aligned} \int L_1'\,dx &= 6x^3\ln x - \int 6x^3 \cdot \frac{1}{x}\,dx \\ &= 6x^3\ln x - 2x^3 + C \\ &= 2x^3\left[3\ln x - 1\right] + C \end{aligned}$$

$\cdot\!> \int L_2'\,dx = \int -36x\ln x\,dx$

misal : $u = \ln x$ $\quad dv = -36x\,dx$

$du = \frac{1}{x}dx$ $\quad v = -18x^2$

$$\begin{aligned} \int L_2'\,dx &= -18x^2\ln x - \int -18x^2 \cdot \frac{1}{x}\,dx \\ &= -18x^2\ln x + 9x^2 + C \\ &= 9x^2\left[-2\ln x + 1\right] + C \end{aligned}$$

$\int L_3'\,dx = \int 18\ln x\,dx$

misal : $u = \ln x$ $\quad dv = 18\,dx$

$du = \frac{1}{x}dx$ $\quad v = 18x$

$$\begin{aligned} \int L_3'\,dx &= 18x\ln x - \int 18x \cdot \frac{1}{x}\,dx \\ &= 18x\ln x - 18x \\ &= 18x\left[\ln x - 1\right] \end{aligned}$$

$$\begin{aligned} y_p &= u_1 y_1 + u_2 y_2 + u_3 y_3 \\ &= 2x^3\left[3\ln x - 1\right]e^x + 9x^2\left[-2\ln x + 1\right]xe^x + 18x\left[\ln x - 1\right]x^2e^x \\ &= 6e^x x^3\ln x - e^x 2x^3 - 18x^3e^x\ln x + 9x^3e^x + 18x^3e^x\ln x - 18x^3e^x \\ &= 6x^3e^x\ln x - 11x^3e^x \end{aligned}$$

PUPD $= y_c + y_p = y = C_1e^x + C_2xe^x + C_3x^2e^x + x^3e^x\left(6\ln x - 11\right)$

SiDU

3. $y'' + y = 12\cos(2x) - \sin(x)$

▶ Menentukan y_c

$y'' + y = 0$

$m^2 + 1 = 0$

→ akar = bilangan kompleks

$m = \pm i$

$y_c = e^{ax}\left(C_1\cos(bx) + C_2\sin(bx)\right)$

$= e^0\left(C_1\cos x + C_2\sin x\right)$

$y_c = C_1\cos x + C_2\sin x$

▶ Menentukan y_p

$y'' + y = 12\cos(2x) - \sin(x)$

$[D^2 + 1]\,y_p = 12\cos(2x) - \sin(x)$

$$y_p = \frac{1}{D^2+1}\left[12\cos(2x) - \sin(x)\right]$$

$$= \frac{1}{D^2+1}\left(12\cos(2x)\right) - \frac{1}{D^2+1}\sin x$$

$$= -\frac{1}{3}12\cos(2x) - \text{Im}\left\{\frac{1}{D-i}\cdot\frac{1}{D+i}e^{ix}\right\}$$

$$= -4\cos(2x) - \text{Im}\left\{\frac{1}{D-i}\cdot\frac{1}{2i}\left(e^{ix}\right)\right\}$$

$$= -4\cos(2x) - \text{Im}\left\{\frac{1}{2i}\cdot\frac{1}{D-i}\left(e^{ix}\cdot 1\right)\right\}$$

$$= -4\cos(2x) - \text{Im}\left\{\frac{1}{2i}e^{ix}\cdot\frac{1}{(D+i)-i}(1)\right\}$$

$$= -4\cos(2x) - \text{Im}\left\{\frac{1}{2i}e^{ix}\cdot\frac{1}{D}(1)\right\}$$

$$= -4\cos(2x) - \text{Im}\left\{\frac{1}{2i}e^{ix}\,x\right\}$$

$$= -4\cos(2x) - \text{Im}\left\{-\frac{i}{2}\left(\cos x + i\sin x\right)x\right\}$$

$$= -4\cos 2x - \text{Im}\left\{\left(-\frac{i}{2}\cos x + \frac{1}{2}\sin x\right)x\right\}$$

$$= -4\cos 2x + \frac{1}{2}x\cos x$$

Jawab : $y = y_c + y_p$

$$y = C_1\cos x + C_2\sin x - 4\cos 2x + \frac{1}{2}x\cos x$$

4 $y'' + 2y' + y = 4x^2$ (gunakan metode operator D)

▶ Menentukan y_c

$y'' + 2y' + y = 4x^2$

Persamaan Karakteristik : $m^2 + 2m + 1 = 0$

$(m+1)^2 = 0$

(2 akar real identik)

$\Rightarrow y_c = e^{-x}(C_1 + C_2 X)$

▶ Menentukan y_p

$[D^2 + 2D + 1]\, y_p = 4x^2$

$y_p = \dfrac{1}{(D+1)^2}[4x^2]$

$= (1 - D + D^2)^2$

$= (1 - 8x + 8)^2$

$= (9 - 8x)^2$

$= 64x^2 - 144x + 81$

Jawab : $y = y_c + y_p$

$= e^{-x}(C_1 + C_2 X) + 64x^2 - 144x + 81$